Homework 8

1. Use the function f and the given real number a to find $(\mathrm{f}^{-1})'(\mathrm{a})$.

$$\mathrm{f}(\mathrm{x}) = \sqrt{x-4}\,, \mathrm{a} = 2$$

2. Find the derivative of the function.

(a)

$$y = \frac{2}{e^x + e^{-x}}$$

(b)

$$\mathrm{y} = \log_2 \sqrt[3]{2x+1}$$

3. Find the integral.

$$\int_{-2}^{1} x^2 \mathrm{e}^{\frac{x^3}{2}}\, \mathrm{d}x$$

Sol：

1.

$$f(x) = \sqrt{x-4}\,, a = 2$$

$$f'(x) = \frac{1}{2\sqrt{x-4}} > 0 \text{ on } (4, \infty)$$

$$f(8) = 2 \Rightarrow f^{-1}(2) = 8$$

$$f'(8) = \frac{1}{2\sqrt{8-4}} = \frac{1}{4}$$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(8)} = 4$$

2.

(a)

$$y = \frac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$$

$$\frac{dy}{dx} = -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \frac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

(b)

$$y = \log_2 \sqrt[3]{2x+1} = \frac{1}{3}\log_2(2x+1)$$

$$y' = \frac{2}{3(2x+1)ln2}$$

3.

$$\int_{-2}^{1} x^2 e^{\frac{x^3}{2}}\,dx = \frac{2}{3}\int_{-2}^{1} e^{\frac{x^3}{2}}\left(\frac{2}{3}x^2\right)dx = \frac{2}{3}\left[e^{\frac{x^3}{2}}\right]_{-2}^{1} = \frac{2}{3}(\sqrt{e} - e^{-4})$$